Wireless Broadcast Using Network Coding

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Abstract

Traditional approaches to transmit information reliably over an error-prone network employ either Forward Error Correction (FEC) or retransmission techniques. In this paper we propose some network coding schemes to reduce the number of broadcast transmissions from one sender to multiple receivers. The main idea is to allow the sender to combine and retransmit the lost packets in a certain way, so that with one transmission, multiple receivers are able to recover their own lost packets. For comparison, we derive a few theoretical results on the bandwidth efficiencies of the proposed network coding and traditional Automatic Repeat-reQuest (ARQ) schemes. Both simulations and theoretical analysis confirm the advantages of the proposed network coding schemes over the Automatic Repeat-reQuest (ARQ) ones.

I. Introduction

Broadcast is a mechanism for disseminating identical information from a sender to multiple receivers. It is widely employed in many applications, ranging from satellite communications to Wireless Local Area Network (WLAN). Reliable broadcast requires that every receiver must receive the correct information sent by the sender. When the communication channels between a sender and receivers are lossy, some appropriate error control schemes must be used to provide reliable transmissions. Depending on applications, these schemes can be classified into two main approaches: Automatic Repeat ReQuest (ARQ) and Forward Error Correction (FEC).

Using the ARQ approach, the sender may have to rebroadcast the lost packet to all the receivers, even though there may be only one receiver that did not receive that packet correctly. The ARQ approach assumes that a feedback channel is available so that the receiver can communicate to the sender on whether or not it receives the correct data. On the other hand, using the pure FEC approach, the sender generates some redundancies, then broadcasts both redundant and original information to the receivers [1]. If the amount of lost data is sufficiently small (less than the redundant data), a receiver can recover the lost data using some decoding schemes.

For satellite TV applications, the TV signals are broadcast from a satellite to potentially millions of TVs, making the probability of any TV not receiving the correct signal at any time close to 1. Therefore, it is extremely inefficient to employ an ARQ protocol for retransmissions since most of the bandwidth will be used for retransmissions. Furthermore, the lack of TV-to-satellite channels makes the retransmission approach infeasible. In these scenarios, it is preferable to employ a pure FEC technique.

In other settings, e.g., WLAN, there are relatively few devices and the communication channel is relatively reliable. Therefore, the retransmission approach may be more bandwidth-efficient than that of the FEC approach since redundant information is not added in every transmission. Furthermore, in practice, FEC alone cannot guarantee reliable delivery due to a non-zero chance that a receiver may not be able to recover the data from a single transmission.

That said, this paper explores the efficient retransmission-based broadcast schemes in single-hop wireless networks. Efficient schemes can be used in WLAN to reduce the time required to copy a large file from one computer to multiple computers simultaneously. It can also be used to broadcast music or important announcements in multiple rooms in a small building, even though a highly reliable audio signal is probably not needed in most situations. In addition, an efficient retransmission-based wireless broadcast scheme might be beneficial to the emerging wireless standard WiMAX (Worldwide Interoperability for Microwave Access). WiMAX aims to enable wireless data transmissions over long distances, and can potentially provide wireless broadband access as an alternative to cable and DSL. In this WiMAX setting, at any point in time, a few homes may want to watch the same broadcast digital TV program through the Internet, i.e., these homes may want to receive the same TV signals. Therefore, a WiMAX broadcast station can view the data delivery as multiple wireless broadcast sessions (TV channels) with each session involving a few homes. By optimizing these wireless broadcast sessions, the wireless bandwidth can be efficiently used. To this end, we propose some broadcast schemes that combine network coding and retransmission to utilize bandwidth efficiently.

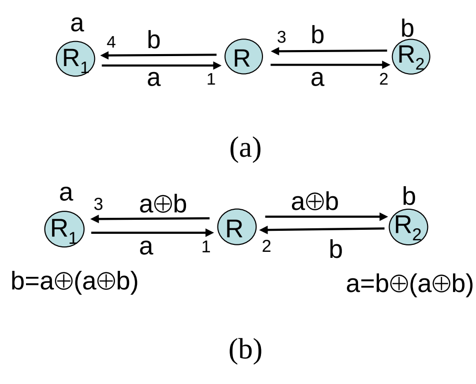

Fig. 1. *Illustration of a traditional store and forward network (a) and a network coding-based network (b).*

The recently introduced network coding theory was the basis for many bandwidth efficient transmission schemes in wireless networks. Informally, network coding refers to the ability of a node in the network to encode the incoming data appropriately before sending these coded data to the next node. The ability to recode the data at the intermediate nodes results in a substantial bandwidth improvement over that of a traditional store and forward network [2]–[6]. Notably, network coding techniques have been applied to increase bandwidth efficiency in wireless ad hoc networks [7]–[11]. As an illustration, Fig. 1 shows an example of packet exchange between nodes R_1 and R_2 via node R in a wireless ad hoc network.

As shown in Fig. 1 (a), without network coding, node R simply relays packet a from R_1 to R_2 and packet b from R_2 to R_1. As a result, the total number of transmissions required for R_1 and R_2 to exchange their packets is 4. On the other hand, when network coding is used, the intermediate node R is allowed to generate and send a new packet out, based on the packets it receives from nodes R_1 and R_2. As shown in Fig. 1 (b), since both nodes R_1 and R_2 can hear the transmission from R, R can generate a new packet by XORing the bits in packets a and b, then broadcasts this new packet $a \oplus b$ to both R_1 and R_2. Upon receiving $a \oplus b$, R_1 can recover b as $a \oplus (a \oplus b)$, R_2 can recover a as $b \oplus (a \oplus b)$. Clearly, in this case, only 3 transmissions are required for R_1 and R_2 to exchange their packets.

Instead of employing network coding for packet exchange in wireless ad hoc networks, our proposed schemes are designed for broadcast in single-hop wireless networks. The main idea for the proposed schemes is based on the observation that, at a certain point in time, many receivers may have disjoint lost packets. Thus, the sender may XOR these lost packets together and broadcast it to all the receivers. Upon receiving this XOR packet, a receiver will be able to recover its lost packet by XORing the XOR packet with the certain packets that it has received previously. As such, one transmission from the sender will enable multiple receivers to recover their lost packets, thus efficiently utilizing the wireless bandwidth. We will elaborate on how to do this shortly and show that this approach can reduce the transmission bandwidth significantly. We note that part of this work has been presented in [12] and [13].

The organization of our paper is as follows. We first discuss related work in Section II. In Section III, we describe different *retransmission-based* broadcast schemes with and without network coding. We then analyze their performances in terms of bandwidth utilization in Section IV. In particular, we derive a few results that show the reduced transmission bandwidth when network coding is employed under different network conditions. Section V presents the simulation results that confirm our theoretical predictions.

II. Related Work

Wireless broadcast is a well-explored problem. In his seminal work, Cover [14] modeled a broadcast channel as multiple binary channels, each with a given channel capacity. He found the lower and upper bounds on the capacity regions of jointly achievable transmission rates. Subsequently, there has been much research on using broadcast bandwidth efficiently. Recently, network coding has been applied successfully to many wireless broadcast and multicast applications. Lun *et al.* [15] proved that the problem of minimum-energy multicast in infrastructureless networks can be solved exactly in polynomial time when employing network coding. This is in contrast with the traditional routing approaches in [16]–[18] that result in non-polynomial time solutions. In addition, Li *et al.* [19], [20] proved that network coding could provide some benefits over the non-network coding approaches. Lun *et al.* [21] showed a capacity-approaching coding scheme for unicast or multicast over lossy packet networks, in which all nodes perform opportunistic coding by constructing the encoded packets with random linear combinations of previously received packets.

Our work is rooted in the recent development of network coding for wireless ad hoc networks [7], [22], [23], [10]. In [7], Wu *et al.* proposed the basic scheme that uses XOR of packets to increase the bandwidth efficiency of a wireless mesh network. In [22], Katti *et al.* implemented a XOR-based scheme in a wireless mesh network and showed a substantial bandwidth improvement over the current approach. Unlike existing approaches, the focus of our work is on the analysis of the *reliable* wireless broadcast problem in a single-hop wireless network such as WLAN or WiMAX networks. Eryilmaz *et al.* [24] also recently proposed a similar model. In this work, Eryilmaz *et al.* employed a *random network coding* scheme for multiple users downloading a single file or multiple files from a wireless base station. Rather than using XOR operations, their scheme encodes every packet using coefficients taken randomly from a sufficiently large finite field [25], [26]. This scheme guarantees that the receivers can decode the original data with high probability. Another work somewhat related to ours is that of Ghaderi *et al.* [27]. In [27], the authors analyzed the reliability benefit of network coding for reliable multicast by computing the expected number of transmissions using link-by-link ARQ compared to network coding.

Last but not least, there exist standard error control techniques for reliable wireless transmission that employ either FEC, ARQ, or hybrid-ARQ [28]. For example, A. Shiozaki *et al.* [29] and M. Nakamura *et al.* [30] investigated hybrid error control broadcast models which incorporate the ARQ with FEC techniques to achieve higher throughputs to all receivers.

III. Broadcast Schemes

To describe our proposed schemes, we make the following assumptions for all the broadcast schemes:

1) There are one sender and $M > 1$ receivers.
2) Data is sent in packets, and each packet is sent in a time slot of fixed duration.
3) The sender has access to the information on packet losses of all the receivers at any time slot. This can be accomplished through the use of positive and negative acknowledgments (ACK/NAKs). For simplicity, we assume that all the ACK/NAKs are instantaneous, i.e. the sender knows (a) whether or not a packet is lost and (b) the identity of the receiver with the lost packet instantaneously. This implicitly assumes that ACK/NAKs are never lost. This assumption is not critical since one can easily incorporate the delay and bandwidth used by ACK/NAKs into the analysis. One can also consider an alternative view in which, if an ACK or NAK is lost, the corresponding data packet is also considered lost. Thus, the assumption of reliable ACK/NAK messages can be relaxed by changing the packet loss probabilities at the receivers to reflect the loss rates in both data and feedback channels.
4) Packet loss at a receiver i follows a Bernoulli trial with parameter p_i. This model is clearly insufficient to describe many real-world scenarios. However, this model is only intended for capturing the essence of wireless broadcast. One can develop a more accurate model, at the cost of complicated analysis. In fact, we will provide some results when using a slightly more accurate model that reflects the correlated losses among the receivers in Section IV-C. We will also provide simulation results using a simple two-state Markov model to characterize packet losses.

A. Broadcast Schemes without Network Coding

Scheme A *(Memoryless receiver)*. In this scenario, a receiver sends a NAK immediately whenever there is a packet loss in the current time slot, regardless of whether it has received this packet correctly in some previous time slots (hence memoryless). This situation arises when a receiver receives a correct packet, but this packet was lost at some other receivers at some previous time slots. Hence, the sender has to retransmit this packet. If this packet is now lost in the current time slot, a memoryless receiver would automatically request a retransmission, even though it has previously received that packet. This scheme is clearly suboptimal in terms of bandwidth utilization as it implies that the sender has to resend a packet until *all* the receivers receive this packet correctly and *simultaneously*.

Scheme B *(Typical ARQ scheme)*. In this scenario, the receiver sends a NAK immediately only if there is a packet loss in the current time slot and this packet has not been received correctly in any previous time slot. This scheme is clearly superior to scheme A in terms of bandwidth utilization since it never requests a retransmission for a packet that it already received successfully.

B. Broadcast Schemes with Network Coding

Scheme C *(Time-based retransmission)*. In this scheme, the receiver's protocol is similar to that of the receiver in scheme B in that, it sends the NAK immediately if it does not receive a packet correctly. However, the sender does not retransmit the lost packet immediately when it receives a NAK. Instead, the sender maintains a list of lost packets and their corresponding receivers for which their packets are lost. The sender waits until N packets have been transmitted before any retransmission takes place. During the retransmission phase, the sender forms a new packet by XORing a maximum set of the lost packets from different receivers before retransmitting this combined packet for all the receivers. The *combined* packets may be lost during the retransmission, and these packets will be retransmitted until all the receivers receive this packet. The sender keeps sending out the *combined* packets until there are no more lost packets on the list, it then resumes the transmission of a different set of packets.

Upon successfully receiving a *combined* packet, a receiver is able to recover its lost packet by XORing this combined packet with an appropriate set of previously successful packets. The information on choosing this appropriate set of packets is included in the packets sent by the sender. To illustrate this, Fig. 2 shows a pattern of lost packets (denoted by the crosses) for two receivers R_1 and R_2. The combined packets are $a_1 \oplus a_3$, $a_4 \oplus a_5$, a_7, a_9, where a_i denotes the i^{th} packet. Note that, if packet $a_1 \oplus a_3$ is not received

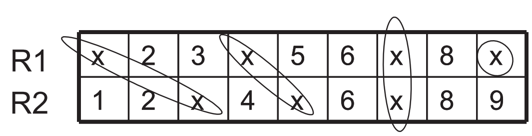

Fig. 2. *Combined packets for time-based retransmission:* $a_1 \oplus a_3$, $a_4 \oplus a_5$, a_7, a_9; $N = 9$

correctly at any receiver, this packet is retransmitted until all the receivers receive this packet correctly, but might not be simultaneously. Receiver R_1 recovers packet a_1 as $a_3 \oplus (a_1 \oplus a_3)$. Similarly, receiver R_2 recovers packet a_3 as $a_1 \oplus (a_1 \oplus a_3)$. When the same packet loss occurs at both receivers R_1 and R_2, the encoding process is not needed and the sender just has to retransmit that packet alone. Note that, the sender has to include some bits to indicate to a receiver which set of packets it should use for XORing. Assuming that all the retransmissions are correctly received at all the receivers at the first attempt, then clearly the number of retransmissions for this scheme is only 4 while it is 6 for scheme B.

Scheme D *(Improved time-based retransmission).* Scheme C is suboptimal because the sender has to retransmit the same combined packet even though some receivers may receive it. An improved scheme is to have the sender dynamically changes the combined packets based on what the receivers have received. For example, Fig. 3 shows the same pattern of lost packets as in the previous scenario. Now, suppose the packet $a_1 \oplus a_3$ is lost at receiver R_2, but is received correctly at receiver R_1. In this case, instead of retransmitting packet $a_1 \oplus a_3$, the sender can transmit packet $a_3 \oplus a_4$. Clearly, on average, the number of transmissions can be further reduced using this scheme.

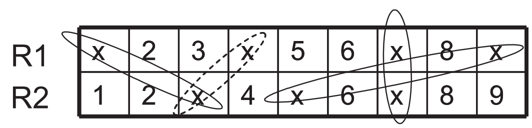

Fig. 3. *Combined packets for improved time-based retransmission:* $a_1 \oplus a_3$, $a_3 \oplus a_4$, $a_5 \oplus a_9$, a_6; $N = 9$.

Remarks: Note that a larger buffer size N results in better bandwidth efficiency. However, it may incur an unnecessary long delay for some packets. This may be acceptable for file transfer, but may not be suitable for multimedia applications. Choosing an optimal value for N for the multimedia applications with certain delay requirements is beyond the scope of this paper. However, we envision that a good scheme is one that dynamically changes the value of N based on the current network conditions and the application delay requirement. When $N = 1$, the network coding scheme reduces to the scheme B. In the next section, we derive a few theoretical results on transmission bandwidths for different schemes with infinite and finite buffer sizes.

IV. Transmission Bandwidth Analysis

We define the transmission bandwidth as the average number of transmissions required to successfully transmit a packet to all the receivers. Let η_A, η_B, η_C, and η_D denote the transmission bandwidths using schemes A, B, C, and D, respectively. Let M denote the number of receivers, and p_i denote the packet loss probability of receiver i. We first discuss the non-network coding schemes A and B.

A. Non-Network Coding Schemes A and B

We begin with a special case where there are only two receivers with the packet loss probabilities of p_1 and p_2. We have the following results:

Proposition 4.1: The transmission bandwidth of scheme A with two receivers is

$$\eta_A = \frac{1}{(1-p_1)(1-p_2)} \tag{1}$$

and using scheme B is

$$\eta_B = \frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1 p_2}. \tag{2}$$

Proof: For scheme A, the proof is simple. As described in Section III, the sender has to retransmit the packets until both receivers receive the correct packets simultaneously. Since the packet loss is independent and uncorrelated between the receivers (Bernoulli trial), the number of transmission attempts before both receivers correctly receive the data follows the geometric distribution with the parameter $(1-p_1)(1-p_2)$. Therefore, the average number of transmissions per successful event is $\frac{1}{(1-p_1)(1-p_2)}$.

For scheme B, let X_1, X_2 be the random variables denoting the numbers of attempts to successfully deliver a packet to R_1 and R_2, respectively. Then, the number of transmissions needed to successfully deliver a packet to both receivers is the random variable $Y = \max\{X_1, X_2\}$. We have

$$P[Y \le k] = P\left[\max\{X_1, X_2\} \le k\right] = \prod_{i=1}^{2} P[X_i \le k] = \prod_{i=1}^{2}(1-p_i^k). \tag{3}$$

Therefore,

$$P[Y = k] = \prod_{i=1}^{2}(1-p_i^k) - \prod_{i=1}^{2}(1-p_i^{k-1}) \tag{4}$$

and the average number of transmissions per successful packet is

$$\begin{aligned} \eta_B &= E[Y] = \sum_{k=1}^{\infty} k\left(\prod_{i=1}^{2}(1-p_i^k) - \prod_{i=1}^{2}(1-p_i^{k-1})\right) \\ &= \sum_{k=1}^{\infty} k(p_1^{k-1}-p_1^k) + \sum_{k=1}^{\infty} k(p_2^{k-1}-p_2^k) + \sum_{k=1}^{\infty} k(p_1^k p_2^k - p_1^{k-1}p_2^{k-1}) \\ &= \frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1 p_2}. \end{aligned} \tag{5}$$

■

Theorem 4.1: The transmission bandwidth of scheme A with M receivers is

$$\eta_A = \frac{1}{\prod_{i=1}^{M}(1-p_i)} \tag{6}$$

and using scheme B is

$$\eta_B = \sum_{i_1, i_2, \ldots, i_M} \frac{(-1)^{i_1+i_2+\ldots i_M - 1}}{1 - p_1^{i_1} p_2^{i_2} \ldots p_M^{i_M}}, \tag{7}$$

where $i_1, i_2, ..., i_M \in \{0, 1\}$, $\exists i_j \neq 0$.

Proof: For scheme A, using the same argument for two receivers, the number of transmissions before all M receivers correctly receive a packet follows the geometric distribution with the parameter $\prod_{i=1}^{M}(1-p_i)$. Therefore, the average number of transmissions per successful packet is $\eta_A = \frac{1}{\prod_{i=1}^{M}(1-p_i)}$.

For scheme B, let $R_1, R_2, ..., R_M$ denote the receivers with the corresponding packet loss probabilities $p_1, p_2, ..., p_M$, respectively. The number of transmissions needed to successfully deliver a packet to all receivers is the random variable $Y = \max_{i \in \{1,..,M\}}\{X_i\}$, where X_i is the random variable denoting the number of attempts to successfully deliver a packet to R_i. We know that $P[Y \leq k] = P\left[\max_{i \in \{1,..,M\}}\{X_i\} \leq k\right] = \prod_{i=1}^{M}(1-p_i^k)$. Therefore,

$$P[Y = k] = P[Y \leq k] - P[Y \leq k-1] = \prod_{i=1}^{M}(1-p_i^k) - \prod_{i=1}^{M}(1-p_i^{k-1}). \tag{8}$$

Thus the average number of transmissions per successful packet is

$$\begin{aligned}
\eta_B &= E[Y] = \sum_{k=1}^{\infty} kP[Y=k] = \sum_{k=1}^{\infty} k\left(\prod_{i=1}^{M}(1-p_i^k) - \prod_{i=1}^{M}(1-p_i^{k-1})\right) \\
&= \sum_{k=1}^{\infty} k\Big((-p_1^k + p_1^{k-1}) + ... + (-p_M^k + p_M^{k-1}) + (-1)^1((p_1^k p_2^k - p_1^{k-1}p_2^{k-1}) + ... \\
&+ (p_{M-1}^k p_{M-2}^k - (p_{M-1}^{k-1} p_{M-2})^{k-1}) + ... + (-1)^M (p_1^k p_2^k ... p_M^k - p_1^{k-1} p_2 ... p_M^{k-1})\Big) \\
&= \sum_{i_1, i_2, ..., i_M} \frac{(-1)^{i_1+i_2+...+i_M-1}}{1 - p_1^{i_1} p_2^{i_2} ... p_M^{i_M}},
\end{aligned} \tag{9}$$

where $i_1, i_2, ..., i_M \in \{0, 1\}$ and $\exists i_j \neq 0$.

■

Note that, with $p_1 = p_2 = ... = p_M = p$,

$$\eta_B = \sum_{k=1}^{M} \frac{(-1)^{k-1}\binom{M}{k}}{1-p^k}. \tag{10}$$

B. Network Coding Schemes C and D

Unlike the schemes A and B, scheme C has one additional parameter, namely, the size of the buffer used to maintain a list of receivers and their corresponding lost packets. When a small buffer is used, there may not be sufficiently many lost packets for generating the combined packets, which can reduce the bandwidth efficiency. On the other hand, when a large buffer is used, the bandwidth efficiency improves at the expense of increased delays for some packets. This approach is acceptable for file transfer applications. We now provide an asymptotic result when the buffer size N and the number of packets to be sent T are sufficiently large. Since it is not beneficial to have $N > T$, we assume $T = N$ and N is sufficiently large. We have the following results for two receivers.

Proposition 4.2: The transmission bandwidth of scheme C with two receivers where $p_1 \leq p_2$ and N is sufficiently large is

$$\eta_C = 1 + \frac{p_1}{1-p_1} + \frac{p_2}{1-p_2} - \frac{p_1}{1-p_1 p_2}. \tag{11}$$

Proof: The key to our proof is the following observation. The transmission bandwidth depends on how many pairs of lost packets one can find in order to generate the combined packets. When the number of packets to be sent is sufficiently large, the probability that the number of lost packets at the receiver R_1 is smaller than that of receiver R_2 is arbitrarily close to 1. Furthermore, the average numbers of lost packets for R_1 and R_2 are Np_1 and Np_2, respectively. This implies that on average, one can combine Np_1 pairs of lost packets since $Np_1 \leq Np_2$. As a result, there are $Np_2 - Np_1$ lost packets from R_2 that need to be retransmitted alone. Therefore, the total number of transmissions required to successfully deliver all N packets to two receivers is simply

$$n = N + Np_1 E[X_1] + N(p_2 - p_1)E[X_2], \tag{12}$$

where X_1 and X_2 are the random variables denoting the numbers of transmission attempts before a successful transmission for the combined and non-combined packets. Now, $E[X_2] = \frac{1}{1-p_2}$ since X_2 follows the geometric distribution. From Proposition 4.1, we have

$$E[X_1] = \frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1 p_2}.$$

Replace $E[X_1]$ and $E[X_2]$ in Equation (12) and divide n by N, we arrive at Proposition 4.2. ■

We can generalize the result to M receivers.

Theorem 4.2: The transmission bandwidth of scheme C with M receivers and sufficiently large N is

$$\eta_C = 1 + p_1 \varphi_M + \sum_{j=1}^{M-1} (p_{j+1} - p_j)\varphi_{M-j}, \tag{13}$$

where

$$\varphi_M = \sum_{i_1, i_2, \ldots, i_M} \frac{(-1)^{i_1+i_2+\ldots+i_M-1}}{1 - p_1^{i_1} p_2^{i_2} \ldots p_M^{i_M}} \tag{14}$$

and $i_1, i_2, \ldots i_M \in \{0, 1\}$, $\exists i_j \neq 0$, $p_1 \leq p_1 \leq \ldots \leq p_M$.

Proof: After a sufficiently large number of transmissions N, the number of lost packets at receivers $R_1, R_2, \ldots, R_M$ are $Np_1, Np_2, \ldots, Np_M$, respectively. Since $p_1 \leq p_2 \leq \ldots \leq p_M$, we have $Np_1 \leq Np_2 \leq \ldots \leq Np_M$. We can conceptually count the number of combinations for XORing the lost packets and transmit these packets in different rounds. In particular, in round 1, there are Np_1 lost packets of R_1 that can be combined with the lost packets of $R_2, R_3, \ldots, R_M$. After these combinations, the numbers of lost packets remain for R_1, R_2, R_3, ..., R_M are 0, $N(p_2 - p_1)$, $N(p_3 - p_1)$, ..., $N(p_M - p_1)$, respectively. Next in round 2, the remaining $N(p_2 - p_1)$ lost packets at R_2 are combined with the remaining lost packets at R_3, R_4, ... R_M. Thus, the remaining lost packets for receivers R_1 to R_M are now 0, 0, $N(p_3 - p_2)$,..., $N(p_M - p_{M-1})$. The same reasoning applies until there are no more lost packets. Therefore, the average number of transmissions required to successfully deliver all N packets to all the receivers equals

$$n = N + Np_1\phi_1 + N(p_2 - p_1)\phi_2 + N(p_3 - p_2)\phi_3 + \ldots + N(p_M - p_{M-1})\phi_M, \tag{15}$$

where ϕ_i denotes the average number of transmissions required to successfully transmit a combined packet in round i.

Now, using Theorem 4.1, the average number of transmission attempts in order for all K receivers to correctly receive a packet is

$$\varphi_K = \sum_{i_1,i_2,...i_K} \frac{(-1)^{i_1+i_2+...+i_K-1}}{1-p_1^{i_1}p_2^{i_2}...p_K^{i_K}}, \tag{16}$$

where $i_1, i_2, ..., i_K \in \{0,1\}$, $\exists i_j \neq 0$, and $p_1 \leq p_2 \leq ... \leq p_K$. Set $\phi_i = \varphi_{M+1-i}$ and divide n by N, the proof follows directly.

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Theorem 4.3: The transmission bandwidth of scheme D with M receivers and sufficiently large N is

$$\eta_D = \frac{1}{1-\max_{i\in\{1,..,M\}}\{p_i\}}. \tag{17}$$

Proof: We begin with the case of two receivers. Without loss of generality, we assume that $p_1 \leq p_2$. As discussed in Section III, the combined packets in scheme D are dynamically formed based on the feedback from the receivers. If a combined packet is correctly received at one receiver, but not at the other, a new combined packet is generated to ensure that the receivers with the correct packet will be able to obtain the new data using the new combined packet. This implies that, in the long run, the number of losses will be dominated by the number of losses at the receiver with the largest error probability (R_2). Therefore, the total number of transmissions to successfully deliver N packets to two receivers equals the number of transmissions to successfully deliver N packets to R_2 alone, i.e. $\frac{N}{1-p_2}$ or $\frac{N}{1-max\{p_1,p_2\}}$. Using a similar argument, we can generalize this result to the case with M receivers:

$$n = \frac{N}{1-\max_{i\in\{1,..,M\}}\{p_i\}}. \tag{18}$$

Therefore, the transmission bandwidth is

$$\eta_D = \frac{n}{N} = \frac{1}{1-\max_{i\in\{1,..,M\}}\{p_i\}}. \tag{19}$$

■

For many real-time applications, it is necessary to reduce to the packet delay. This implies that the retransmission of lost packets from the sender to the receivers must be done promptly, i.e., N should be sufficiently small. However, by doing so, the chance of combining the lost packets decreases. Thus, we want to quantify the bandwidth efficiency for scheme D with finite buffer size N. We have the following theorem:

Theorem 4.4: The transmission bandwidth of scheme D with M receivers and buffer size N is

$$\eta_D^N = \frac{\sum_{k=N}^{\infty} k\left(\prod_{j=1}^{M}\sum_{i=0}^{k-N} Q_{ij} - \prod_{j=1}^{M}\sum_{i=0}^{k-N-1} Q_{ij}\right)}{N}, \tag{20}$$

$where$

$$Q_{ij} = \begin{cases} p_j^i(1-p_j)^N \sum_{l=1}^{i} \binom{N}{l}\binom{i-1}{l-1} & \text{with } i \leq N \\ p_j^i(1-p_j)^N \sum_{l=1}^{N} \binom{N}{l}\binom{i-1}{l-1} & \text{with } i > N \end{cases}$$

and p_j is the loss probability at receiver R_j.

Proof: The proof is provided in the Appendix. ■

Note that we have not been able to obtain a reasonable closed-form expression for the transmission bandwidth of scheme C with a finite buffer. Thus, we omit the analysis for this case.

C. Receivers with Correlated Loss

The previous results are obtained based on a simple Bernoulli model for packet loss in a wireless medium. In many scenarios, packet losses at different receivers are highly correlated. For example, if two wireless receivers are located closely to each other, and behind an obstacle, then most likely they will have correlated losses. Thus, the assumption on independent packet losses among the receivers is no longer accurate. In this section, we would like to investigate the performance gain of network coding schemes under such scenarios. In particular, we first assume that packet losses at different receivers in a given time slot can be correlated, and their loss probabilities are given by a joint probability. Second, we assume that packet losses in different time slots are uncorrelated. We now have the following results on transmission bandwidth for M receivers with correlated losses.

Theorem 4.5: The transmission bandwidth for an M-receiver scenario with correlated losses and sufficiently large buffer using scheme B is

$$\eta_{cor}^{B} = \sum_{k=1}^{\infty} kP\left[Y_M = k\right] \tag{21}$$

and using scheme C is

$$\eta_{cor}^{C} = 1 + \sum_{l=0}^{M-1} \sum_{k=1}^{\infty} k(p_{l+1} - p_l)P\left[Y_{M-l} = k\right], \tag{22}$$

where $p_1 \leq p_2 \leq ... \leq p_M$, $p_0 = 0$ *and* $P[Y_{M-l} = k]$ *is the probability that the sender needs* k *transmissions to deliver a packet to all* $M - l$ *receivers* $R_l, R_{l+1}, ..., R_M$ *successfully.*

Proof: The proof can be found in the Appendix. ■

The theorem above indicate that to compute the transmission bandwidth, one needs to compute the probabilities $P[Y_M = k]$ and $P[Y_{M-l} = k]$. We show how to compute these probabilities in the Appendix.

The transmission bandwidth with correlated losses in Scheme D is the same as that in the case of independent receivers, i.e.,

$$\eta_{cor}^{D} = \frac{1}{1 - \max_{i \in \{1,..,M\}}\{p_i\}}.$$

This is because, in the long run, regardless of whether the packet losses are correlated or not, the number of transmissions to successfully deliver N packets to M receivers will be dominated by the one with the largest loss probability.

D. Remarks on Network Coding Gain

In the previous section, we analyzed the transmission bandwidths of different schemes. We now define the coding gain of one scheme over the other by the ratio of their transmission bandwidths. In particular, the coding gains of schemes C and D over scheme B for two receivers are

$$G_C = \frac{\eta_B}{\eta_C} = \frac{\frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1p_2}}{1 + \frac{p_1}{1-p_1} + \frac{p_2}{1-p_2} - \frac{p_1}{1-p_1p_2}} \tag{23}$$

and

$$G_D = \frac{\eta_B}{\eta_D} = \frac{\frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1p_2}}{\frac{1}{1-max\{p_1,p_2\}}}. \tag{24}$$

For the case $p_1 = p_2 = p$, Equations (23) and (24) become

$$G_C = \frac{1+2p}{1+p+p^2} \tag{25}$$

and

$$G_D = \frac{1+2p}{1+p}. \tag{26}$$

Note that, when p_1 or p_2 is equal to zero, Equations (23) and (24) indicate no gain for network coding schemes, e.g., $G_C = G_D = 1$. However, this scenario is only true when considering only two receivers. A typical scenario is likely to involve more users with different packet loss rates. Even in the presence of lossless receivers, if there are a few lossy receivers (more than 1), our schemes still provide higher bandwidth efficiency. We will continue this discussion in the next section.

V. Simulation Results and Discussion

We use simulations to (a) verify the analytical derivations for the transmission bandwidths and (b) to set light on the typical performances of different broadcast schemes for real world settings. Instead of using Raleigh fading parameters to characterize the wireless channel, we use packet loss rates which is sufficiently characterize the overall health of the channel. For interested readers, in [31], we provide some analysis of our proposed techniques with a detail consideration on the modulation and channel parameters. However, such analysis is beyond the scope of this paper. Also, our simulations do not take into account the interaction between the MAC protocol and the higher layer protocol such as TCP. Under some settings, this interaction may reduce the coding gain for the proposed schemes. Recently, Dong et al. [32] provide a discussion on possible performance degradation of network coding when TCP is employed in wireless ad hoc network. As such, the authors provide a *loop coding* scheme that improves both network throughput and TCP throughput simultaneously. Modeling such a complex interaction is very useful, however it is beyond the scope of this paper.

That said, the simulations are divided into four categories. In category one, packet losses are assumed to be independent and uncorrelated across the receivers. In category two, packet losses are also assumed independent across the time slots, but they are correlated among the receivers. In both of these categories, we attempt to model a realistic performance of the proposed scheme by using the simulated packet loss rates for the IEEE 802.11 standard as reported in the literature. In particular, the reported packet loss rates (before the MAC protocol retransmissions) range from 0.5% to 20% [33]. Similar packet loss rates are confirmed in [34]. In category three, we model the channel as a two-state Markov chain to capture the burst losses. Finally, in category four, we employ trace-driven simulations. The traces are collected from measurements of packet losses in IEEE 802.11 based network [33]. We now begin with the simulations for category one.

A. Independent and Uncorrelated Packet Loss Model

Fig. 4 shows the simulation and theoretical results on the transmission bandwidths (e.g., the average number of transmissions per successful packet) of schemes A, B, C, and D for the scenario consisting of one sender and two receivers R_1 and R_2 with independent and uncorrelated packet losses. For schemes C and D, we use the buffer size $N = 1000$ packets, which sufficiently simulates an infinite size buffer in this setting. The packet loss probability of R_1 varies as shown on the x-axis while that of R_2 remains at 10%. As seen, the simulation and theoretical curves match all most exactly for all the schemes, verifying the results of our derivations. Furthermore, the number of transmissions per successful packet in scheme D is the smallest while that of scheme A is the largest, which confirms our earlier intuitions about these schemes. We note that although scheme D is slightly more efficient than scheme C, the hardware implementation of scheme D might be little more complex than that of scheme C due to its dynamic selection of the retransmitted packets.

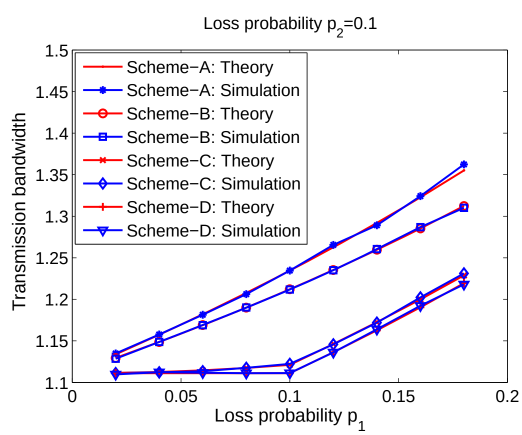

Fig. 4. *Transmission bandwidth versus packet loss probability.*

We now show the coding gains of different schemes. Ideally, scheme A has the worst performance and should be used as the baseline for comparison. However, most wireless devices with limited memory will be able to implement scheme B. Furthermore, scheme B is the traditional ARQ scheme. Therefore, we compare our proposed schemes C and D against scheme B by examining their coding gains over scheme B. Fig. 5 shows the coding gains of schemes C and D as functions of packet loss probability of R_1. It is interesting to note that the gain is largest when both loss probabilities of R_1 and R_2 are equal to each other. This is intuitively plausible as in this special case, the maximum number of lost packet pairs is achieved. In other words, more combined packets can be generated, reducing the number of retransmissions required otherwise.

On the other hand, when two receivers have disparate packet loss rates, e.g., $p_1 = 0.01$, $p_2 = 0.9$, using the network coding techniques, roughly 1% of the combined packets and 89% of individual lost packets must be retransmitted. Since network coding techniques depend on the number of lost packets that can be combined, it would not produce much coding gain in this scenario. At one extremity, if one receiver does not have any packet loss and the other has some non-zero packet loss rates, e.g., 10%, then the performance of the network coding technique is identical to that of scheme B, the traditional ARQ technique, i.e., coding gain over scheme B equals 1.

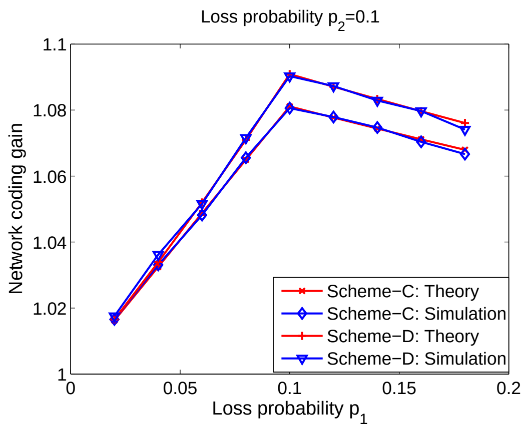

Fig. 5. *Network coding gain versus packet loss probability.*

In addition, the coding gains for schemes C and D over scheme B seem to be the piece-wise linear functions of the loss rate p_1 (with fixed p_2). However, this is simply a coincidence for this range of values for p_1 and p_2. The coding gains of scheme D over B can be easily calculated as $(\frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1 p_2})(1-p_2)$ for the first segment, and $(\frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1 p_2})(1-p_1)$ for the second segment. These functions are clearly not linear functions, but their plots resemble linear plots. Also note that the cusp in the graph is due to the sudden change of $\max\{p_1, p_2\}$ from p_2 to p_1. Recall that for scheme D, the transmission bandwidth is $\frac{1}{1-max\{p_1,p_2\}}$. Since p_2 is fixed and p_1 changes along the x-axis. Therefore, the transmission bandwidth remains constant at $\frac{1}{1-p_2}$ until p_1 exceeds p_2. After this point, the transmission bandwidth starts varying with p_1, creating a cusp in the graphs.

To investigate the effectiveness of our proposed techniques as functions of the number of the receivers, Fig. 6 shows the average number of transmissions required to successfully deliver a packet to all receivers for schemes A, B, C and D. In this scenario, the loss probabilities of all the receivers are set to 0.1. The network coding schemes C and D significantly outperform schemes A and B when the number of receivers is large. As the number of receivers increases, the transmission bandwidths for schemes A and B increase significantly. This is because it is much harder to successfully transmit a packet to all the receivers due to the increase in likelihood that a lost packet can occur at any receiver. For example, if the packet loss rate

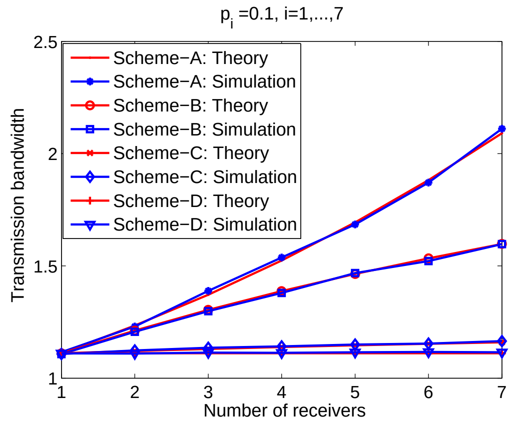

Fig. 6. *Transmission bandwidth versus the number of receivers.*

of receiver R_1 is p_1, then the average number of transmissions required to successfully transmit a packet is $\frac{1}{1-p_1}$. Now if one is required to successfully transmit the same packet also to receiver R_2 with packet loss rate of p_2, then using scheme A, one needs an average of $\frac{1}{(1-p_1)(1-p_2)}$ transmissions. Note that the denominator is the product of terms that are less than 1, which quickly reduces to a small number when the number of receivers increases. As a result, the transmission bandwidth quickly increases. On the other hand, the transmission bandwidth for scheme C increases very slightly and is unchanged for scheme D. As shown in the analysis, the transmission bandwidths of these network coding schemes depend more or less on the receiver with the highest packet loss rate. Since the loss rates are set to 0.1 for all the receivers, we should not expect to see much increase in the transmission bandwidth. In fact, for scheme D, the transmission bandwidth should not increase at all since it is equal to $\frac{1}{1-max\{p_1,...,p_7\}} = \frac{1}{1-0.1} = 1.1$, which is not a function of the number of receivers. Intuitively, even though there are more packet losses with more receivers, using network coding techniques, most of these lost packets can be combined, effectively reducing the total number of retransmissions.

Fig. 7 shows the theoretical and simulated coding gains (over scheme B) for five receivers with different loss probabilities for schemes C and D as a function of packet loss probability p_1 at receiver R_1. The packet loss probabilities at other receivers are set as follows: $p_2 = p_3 = 0$; $p_4 = p_1 + 0.3$; and $p_5 = 0.3$, i.e. there are packet losses at receivers R_1, R_4 and R_5, but not R_1 and R_2. As seen, the network coding gain of scheme D is 15% when $p_1 = 0.1$. Note that, even if there are two receivers without packet loss, our network coding schemes are still better than the traditional retransmission scheme. This is plausible since whenever there are pairs of disjoint packet losses at two or more receivers, the XOR packets are formed and transmitted in the network coding schemes, leading to a better performance.

Up until now, we have shown the results of different schemes under the infinite buffer assumption. In practice, one must use a finite buffer. To characterize the performance of the best scheme (scheme D) with a finite buffer, Fig. 8 shows the transmission bandwidth as a function of the buffer size N for scheme D at $p_1 = p_2 = 0.2$. As expected, as the buffer size increases, the number of opportunities for

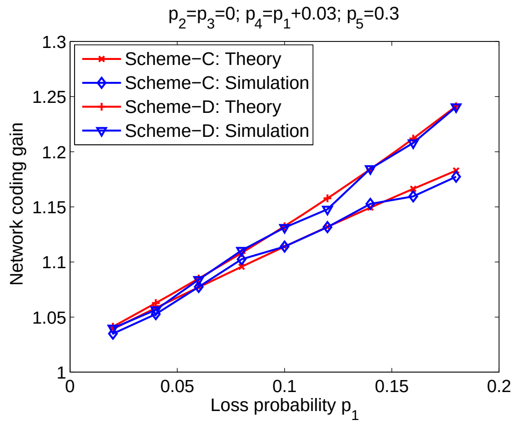

Fig. 7. *Network coding gains versus packet loss probability in a 5-receiver scenario.*

combining lost packets increases, resulting in a smaller number of retransmissions. When the buffer size is large, e.g., more than 40 and $p_1 = p_2 = 0.2$, the transmission bandwidth changes very slightly. This is because such a buffer size is sufficiently large and can be thought of having an infinite value, and thus the transmission bandwidth remains constant according to Theorem 4.3. Under high loss rates, one can use a shorter buffer and still achieve the performance of the scheme with an infinite buffer. On the other hand, one needs to use a larger buffer when packet losses are infrequent to achieve the performance limit. Clearly, using a larger buffer size results in longer delay for certain packets, and may not be acceptable for some real-time applications. An interesting question is how to find an optimal buffer size under the delay constraints. We have addressed this question partially in [35].

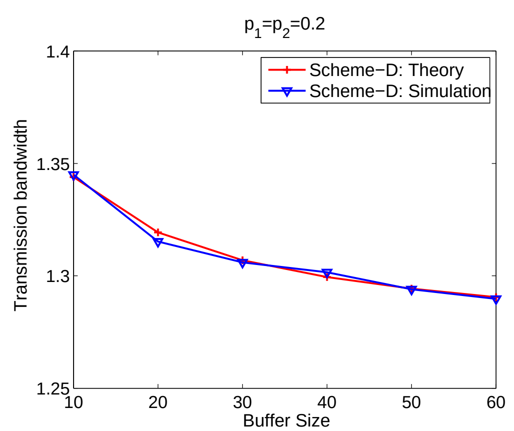

Fig. 8. *Transmission bandwidth versus buffer size for scheme D in a 2-receiver scenario.*

B. Independent but Correlated Packet Loss Model

In this model, the receivers are assumed to have correlated packet losses. In particular, we assume there are two receivers with the following loss characteristics: whenever there is a packet loss at R_1, with probability of 0.7 that packet is also lost at R_2 and whenever a packet is received successfully at R_1, with probability of 0.9 that packet is also received successfully at R_2. Note that both conditional probabilities are above 0.5 which imply positive correlations between successful receptions as well as packet losses at these two receivers. We note that these conditional probabilities can be computed from the given joint probability mass function. Fig. 9 shows the network coding gains versus the packet loss probability for schemes C and D in the case of two receivers with correlated losses. As seen, as the packet loss probability of R_1 increases, the network coding gain also increases. However, the network coding gain for correlated loss receivers is not as large as that of receivers with independent losses. To see why, we consider two correlated loss receivers when $p_1 = 0.1$. This implies that

$$\begin{aligned} p_2 &= P(R_2 = \text{“}x\text{”}|R_1 = \text{“}o\text{”})P(R_1 = \text{“}o\text{”}) + P(R_2 = \text{“}x\text{”}|R_1 = \text{“}x\text{”})(R_1 = \text{“}x\text{”}) \\ &= (1 - P(R_2 = \text{“}o\text{”}|R_1 = \text{“}o\text{”}))(1 - p_1) + P(R_2 = \text{“}x\text{”}|R_1 = \text{“}x\text{”})p_1 \\ &= 0.16 \end{aligned} \quad (27)$$

Now, the coding gain for the correlated loss receivers at this point in Fig. 9 is 1.03 while the coding

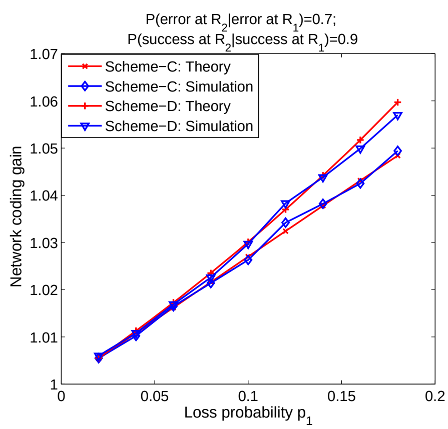

Fig. 9. *Coding gain versus packet loss probability in a scenario with correlated losses.*

gain for the independent loss receivers at the same point ($p_1 = 0.1, p_2 = 0.16$) in Fig. 5 is 1.08. Clearly, network coding techniques are less useful in correlated loss environments. This is intuitively plausible by considering one extremity where packet losses at the receivers are 100% correlated. In this case, the network coding scheme simply reduces to the traditional retransmission scheme since the packet receptions at two receivers are completely identical. Fig. 10 confirms this phenomenon as the correlation between the two receivers increases, the network coding gain reduces.

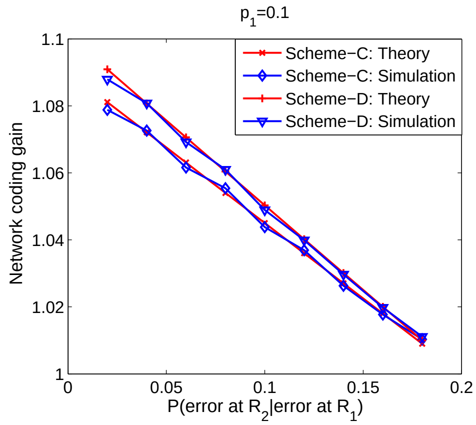

Fig. 10. *Coding gain versus conditional packet loss probability in a scenario with correlated losses.*

C. Two-State Markov Model

We now present the simulation results based on a two-state Markov model for characterizing the bursty packet losses. Using the two-state Markov model, the state of a channel is classified into "bad" and "good" states. When the channel is in the good state, the packet loss probability p_{good} is small, and when it is in the bad state, the packet loss probability p_{bad} is much larger. The channel state changes at each transmission slot with transition probabilities $\alpha = p_{good->bad}, \beta = p_{bad->good}$ as shown in Fig. 11. The stationary probabilities for the channel in the good and bad states are $\pi_{good} = \frac{\beta}{\beta+\alpha}$ and $\pi_{bad} = \frac{\alpha}{\beta+\alpha}$, respectively.

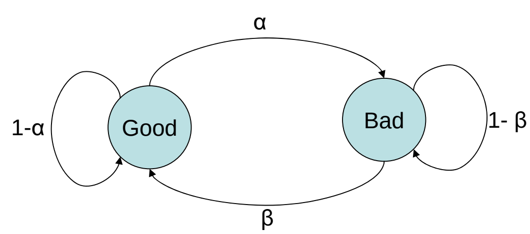

Fig. 11. *Transition of channel states in two-state Markov error model.*

We evaluate the performances of different schemes for a 5-receiver scenario, with each receiver having identical channel conditions. For simplicity, we set β to a constant value while varying α. Fig. 12 shows the transmission bandwidths of different schemes. When $\alpha = 0$, the channel quickly converges and stays in the good state which has very small loss probability. Thus the performances of all schemes are almost identical. As α increases while β is unchanged, the portion of time that the channel has a high loss probability becomes larger. As a result, there are more lost packets, and more combined packets to be transmitted, leading to large performance gaps between network coding schemes (C, D) and non-network

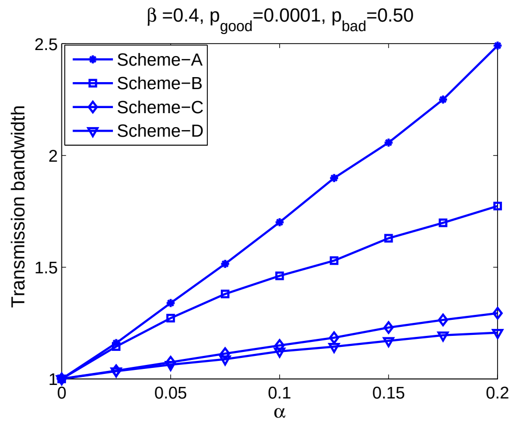

Fig. 12. *Transmission bandwidth versus state transition probability using two-state Markov error model in a 5-receiver scenario.*

coding schemes (A,B). In other words, the transmission bandwidths for network coding schemes do not increase as fast as those of the non-network coding schemes as the channel gets progressively worse.

D. Trace-driven Simulation

We now show the performances our proposed schemes using trace-driven simulations. In particular, Fig. 13 shows the actual packet error rates of the IEEE 802.11 standard for different traces [33] prior to the MAC layer retransmission. Each trace consists of 20,000 packets; they recorded the average packet loss rates during the transmissions from an AP to a receiver. We note that the packet loss rates are substantially large for traces larger than 36. We ignore these traces since they are measured when the sending rate is set to a very high value. Instead, we use traces 0 to 36 to evaluate the performances of our schemes. Since these traces were collected in an experiment involving only a single sender and a single receiver, we need a way to assign traces to multiple receivers in order to simulate broadcast scenarios. That said, to simulate a scenario consisting of 5 receivers, we arbitrarily picked traces 1 to 12 to represent channel conditions for receivers 1 and 2, traces 20 to 31 for receiver 3, traces 15 to 26 for receiver 4, and traces 25 to 36 for receiver 5. Effectively, each receiver experiences 12 different packet loss rates through time, corresponding to 12 different traces with receivers 1 and 2 having identical channel conditions. Fig. 14 shows the transmission bandwidths for different schemes vs trace number. Each point on the graph represents the transmission bandwidth when the packet loss rate for each receiver is taken from its traces in the increasing order. For example, the first point on the graph for scheme A corresponds to the packet loss rates for receivers 1 to 5 taken from traces 1, 1, 20, 14, 25 respectively. As seen in Fig. 14, traces with high loss probabilities result in larger performance improvements for network coding schemes over non-network coding schemes, whereas there are almost no differences among the performances of all schemes for traces with near-zero error rates. This confirms our theoretical analysis.

Remark: We note that the performances of our proposed schemes depend on the packet loss rates, which are functions of many parameters. Among these parameters are the packet size and the bit error rate

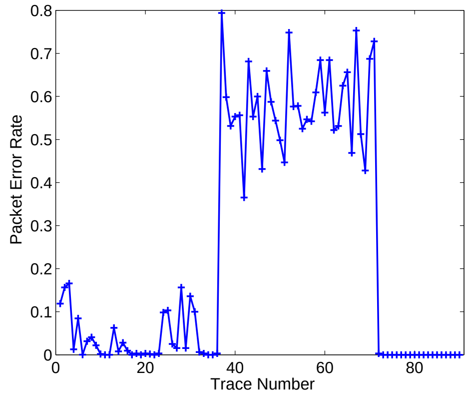

Fig. 13. *Packet error rates for different traces [33].*

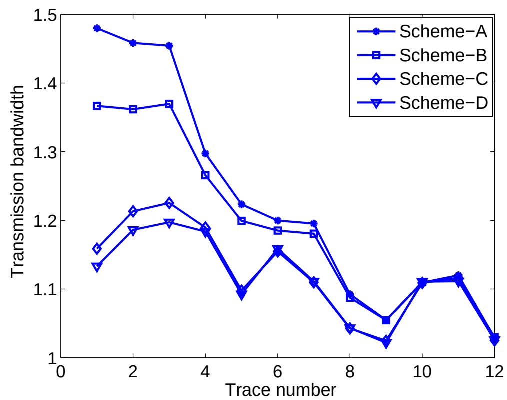

Fig. 14. *Transmission bandwidths for different traces in a 5-receiver scenario.*

of the channel. The packet size is tunable. The bit error rate can be reduced to a desired rate by adding appropriate amount of FEC redundancy. Therefore, one can modify the packet error rate for optimal bandwidth transmission by changing the packet size and the amount of redundancy. Of course, when doing so, one must take into account the redundancy introduced by FEC. Even when an optimal FEC is used for a given channel, we emphasize that our proposed network coding technique with FEC still outperforms a hybrid-ARQ technique that uses the same amount of FEC. This is because adding the same amount of FEC just simply changes the packet loss probabilities for both techniques by an equal amount. Furthermore, we have shown that network coding technique for retransmission is better than the traditional ARQ technique. Therefore, one should expect that a joint network coding and channel coding technique is better than a hybrid-ARQ technique. In [31], we provide some analysis for jointly optimizing network coding and channel coding techniques.

VI. CONCLUSION

In this paper we propose some network coding techniques to increase the bandwidth efficiency of reliable broadcast in a wireless network. Our proposed schemes combine different lost packets from different receivers in such a way that multiple receivers are able to recover their lost packets with one transmission by the sender. The advantages of the proposed schemes over the traditional wireless broadcast are shown through simulations and theoretical analysis. Specifically, we provide a few results on the transmission bandwidths of the proposed schemes under different channel conditions. We are currently investigating a joint approach of network coding, retransmission, and FEC in order to further increase the bandwidth utilization.

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APPENDIX

Proof of Theorem 4.4: We consider the scenario with one sender and one receiver. Let X denote the number of transmissions for the receiver to get N packets successfully. X can be N, $N+1$, $N+2$, ...We compute the probabilities for different values of X.

If exactly N transmissions are required, then there must be no loss during transmitting N packets. We have

$$P[X = N] = \binom{N}{0} p^0 (1-p)^N.$$

If $N+1$ transmissions are required, then there must be only one lost packet and one successful retransmission. We have

$$P[X = N+1] = \left[\binom{N}{1} p(1-p)^{N-1}\right](1-p) = \binom{N}{1} p(1-p)^N.$$

If $N+2$ transmissions are required, then there could be one loss during the transmissions of the first N packets and two retransmissions before the lost packet is successfully received, or there could be two losses during the transmissions of the first N packets and two successful retransmissions, one for each lost packet. We have

$$\begin{aligned} P[X=N+2] &= \left[\binom{N}{1}p(1-p)^{N-1}\right]p(1-p)+\left[\binom{N}{2}p^2(1-p)^{N-2}\right](1-p)^2 \\ &= \left[\binom{N}{1}+\binom{N}{2}\right]p^2(1-p)^N. \end{aligned} \tag{A.28}$$

Similarly, one can show that

$$P[X=N+i]=\begin{cases} p^i(1-p)^N\sum_{l=1}^{i}\binom{N}{l}\binom{i-1}{l-1} & \text{with } i\le N \\ p^i(1-p)^N\sum_{l=1}^{N}\binom{N}{l}\binom{i-1}{l-1} & \text{with } i>N. \end{cases} \tag{A.29}$$

Now, we consider the case of one sender and two receivers.

The number of transmissions using network coding to guarantee that both receivers receive N packets successfully is $Y=\max_{j\in\{1,2\}}\{X_j\}$ where X_j is a random variable denoting the number of transmissions for receiver j to get N packets successfully. Then,

$$P[Y\le k]=\prod_{j=1}^{2}\sum_{i=0}^{k-N}P[X_j=i+N]. \tag{A.30}$$

Next,

$$P[Y=k]=\prod_{j=1}^{2}\sum_{i=0}^{k-N}Q_{ij}-\prod_{j=1}^{2}\sum_{i=0}^{k-N-1}Q_{ij}, \tag{A.31}$$

where $Q_{ij}=P[X_j=N+i]$, which can be computed from (A.29).

The average number of transmissions so that both receivers receive a packet successfully is

$$\eta_D^N=E[Y]=\sum_{k=N}^{\infty}kP\left[Y=k\right]=\sum_{k=N}^{\infty}k\left(\prod_{j=1}^{2}\sum_{i=0}^{k-N}Q_{ij}-\prod_{j=1}^{2}\sum_{i=0}^{k-N-1}Q_{ij}\right). \tag{A.32}$$

Without much difficulty, we can generalize the result for the case with M receivers as

$$E[Y]=\sum_{k=N}^{\infty}k\left(\prod_{j=1}^{M}\sum_{i=0}^{k-N}Q_{ij}-\prod_{j=1}^{M}\sum_{i=0}^{k-N-1}Q_{ij}\right), \tag{A.33}$$

where

$$Q_{ij}=\begin{cases} p_j^i(1-p_j)^N\sum_{l=1}^{i}\binom{N}{l}\binom{i-1}{l-1} & \text{with } i\le N \\ p_j^i(1-p_j)^N\sum_{l=1}^{N}\binom{N}{l}\binom{i-1}{l-1} & \text{with } i>N \end{cases}$$

and p_j is the probability that the packet is lost at receiver R_j.

Proof of Theorem 4.5 Scheme B (outline): We start with the case of two receivers, then generalize to M receivers. Denote X_1 and X_2 the number of attempts needed to successfully deliver a packet to receiver R_1 and R_2 respectively, and $Y=\max_{i\in\{1,2\}}\{X_i\}$. We have

$$P[Y\le k]=\sum_{i=1}^{k}\sum_{j=1}^{k}P[X_1=i,X_2=j]. \tag{A.34}$$

Next,

$$P[Y=k] = \sum_{i=1}^{k}\sum_{j=1}^{k} P[X_1=i, X_2=j] - \sum_{i=1}^{k-1}\sum_{j=1}^{k-1} P[X_1=i, X_2=j]$$
$$= \sum_{i=1}^{k} P[X_1=i, X_2=k] + \sum_{i=1}^{k-1} P[X_1=k, X_2=i]. \quad \text{(A.35)}$$

Number of attempts	1	2	…	i-1	i	i+1	…	k-1	k
R_1	x	x	…	x	o	-	…	-	-
R_2	x	x	…	x	x	x	…	x	o

Fig. 15. *Number of attempts to deliver a packet to both receivers successfully: R_1 needs i while R_2 needs k attempts ("x" and "o" indicate an unsuccessful and successful attempt, respectively).*

Fig. 15 shows that after $X_1 = i$ and $X_2 = k$ attempts ($i \leq k$), a packet is received successfully at R_1 and R_2 respectively. Assume that we know the joint packet loss probability. Let us denote p_{xx}, p_{xo}, p_{ox}, and p_{oo} as the probabilities that a packet is lost at both receivers, a packet is lost at R_1 but received at R_2, a packet is received at R_1 but lost at R_2, and a packet is received at both receivers, respectively. We have

$$P[X_1=i, X_2=j] = \begin{cases} p_{xx}^{i-1} p_{ox} (p_{ox}+p_{xx})^{j-i-1}(p_{oo}+p_{xo}) & \text{with } i \leq j \\ p_{xx}^{j-1} p_{xo} (p_{xo}+p_{xx})^{i-j-1}(p_{oo}+p_{ox}) & \text{with } i > j. \end{cases} \quad \text{(A.36)}$$

Therefore, the average number of transmissions required to send a packet successfully to both receivers is

$$E[Y] = \sum_{k=1}^{\infty}\sum_{i=1}^{k} kP[X_1=i, X_2=k] + \sum_{k=1}^{\infty}\sum_{i=1}^{k-1} kP[X_1=k, X_2=i]$$
$$= \sum_{k=1}^{\infty}\sum_{i=1}^{k} k p_{xx}^{i-1} p_{ox}(p_{ox}+p_{xx})^{k-i-1}(p_{oo}+p_{xo})$$
$$+ \sum_{k=1}^{\infty}\sum_{i=1}^{k-1} k p_{xx}^{i-1} p_{xo}(p_{xo}+p_{xx})^{k-i-1}(p_{oo}+p_{ox}). \quad \text{(A.37)}$$

Now examine the case of M receivers. Let X_i denote the number of attempts for a packet to be received successfully at receiver R_i, $Y_M = \max_{i\in\{1,\ldots,M\}}\{X_i\}$, and $P[j_1, j_2, ..., j_M] = P[X_1 = j_1, X_2 = j_2, ..., X_M = j_M]$. We have

$$P[Y_M=k] = \sum_{j_1,\ldots,j_M \in Z_k} P[j_1, j_2, ..., j_M] - \sum_{j_1,\ldots,j_M \in Z_{k-1}} P[j_1, j_2, ..., j_M], \quad \text{(A.38)}$$

where $Z_k = \{1, ..., k\}$ and $Z_{k-1} = \{1, ..., k-1\}$. Note that Z_k and Z_{k-1} are defined to make k largest among $j_1, j_2, ...j_M$.

Now, we can compute $P[j_1, j_2, ...j_M]$ in terms of the joint packet loss probabilities. Let the vector $(a_1 a_2 ... a_M)$ denote the status reception of a packet at all the receivers; a_h = "o" and $a_h =$ "x'' indicate successful and unsuccessful receptions at receiver R_i, respectively. Let $p_{a_1 a_2 \ldots a_M}$ denote the probability of this event, then:

$$\sum_{j_1,...,j_M \in Z_k} P[j_1, j_2, ..., j_M] = \sum_{j_1,...,j_M \in Z_k} \left(p^{i_1}_{x_1 x_2 ... x_M} p_{o_1 x_1 ... x_M} \prod_{h=1}^{M-1} \left(\sum_{a_h \in \{o_h, x_h\}} p_{a_1 ... a_h x_{h+1} ... x_M} \right)^{l_{h+1} - l_h - 1} \left(\sum_{a_h \in \{o_h, x_h\}} p_{a_1 ... a_h o_{h+1} x_{h+2} ... x_M} \right) \right) \quad \text{(A.39)}$$

where the sequence $\{l_1, l_2, ..., l_M\}$ is is an ascending sorted sequence of $\{j_1, j_2, ..., j_M\}$. For instance, if $\{j_1, j_3, j_2\}$ is the ascending sorted sequence of $\{j_1, j_2, j_3\}$, then then $l_1 = j_1$, $l_2 = j_3$, and $l_3 = j_2$.

The key is to obtain the above equation is to set up a table as shown in Fig. 15 and multiply out the joint probability as it was done in Equation (A.36) for the 2-receiver case.

Given $P[Y_M = k]$, the transmission bandwidth for M receivers with correlated losses using scheme B is

$$\eta^B_{cor} = E[Y] = \sum_{k=1}^{\infty} kP\left[Y_M = k\right] \quad \text{(A.40)}$$

Proof of Theorem 4.5 Scheme C: Consider the 2-receiver scenarios. We use the same notations above and assume that $p_1 \leq p_2$. In the long run, the average number of lost packets at receiver 2 with be larger than that at receiver 1. Then average number of transmissions to successfully deliver a packet to two receivers is

$$\eta^C_{cor} = 1 + p_1 E[max\{X_1, X_2\}] + (p_2 - p_1)E[X_2], \quad \text{(A.41)}$$

where $E[max\{X_1, X_2\}]$ is obtained from equation A.37 and $E[X_2] = \frac{1}{1-p_2}$.

For the general case of M receivers, we also assume that $p_i \leq p_j$ for all $i < j$. Similarly, in the long run, the number of lost packets at receiver i is smaller than that of receiver j. Using the same argument for the case of 2 receivers we can derive the average number of transmissions required to successfully deliver a packet to M receivers as

$$\begin{aligned} \eta^C_{cor} &= 1 + p_1 E[\max_{i \in \{1,..,M\}} \{X_i\}] + (p_2 - p_1)E[\max_{i \in \{2,..,M\}} \{X_i\}] + ... + (p_M - p_{M-1})E[X_M] \\ &= 1 + \sum_{l=0}^{M-1} (p_{l+1} - p_l) \sum_{k=1}^{\infty} kP\left[Y_{M-l} = k\right] \\ &= 1 + \sum_{l=0}^{M-1} \sum_{k=1}^{\infty} k(p_{l+1} - p_l)P\left[Y_{M-l} = k\right], \end{aligned} \quad \text{(A.42)}$$

where $p_0 = 0$ and $P[Y_{M-l} = k]$ is the probability that the sender needs k transmissions to deliver a packet to all $M - l$ receivers $R_l, R_{l+1}, ...,$ and R_M successfully. $P[Y_{M-l} = k]$ can be computed using Equation (A.39). Note that, to shorten the notation, we add a virtual receiver R_0 with $p_0 = 0$.